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ABSTRACT

The study investigated whether (1) instruction providing for experimental manipulation by students was superior to instruction in which teachers perform the manipulations and children observe, and (2) spatial ability was related to mode of instruction. Participating in the 8-week study were 141 students aged 12-15 in seven intact eighth-grade classes. To measure spatial ability, the Card Rotations Test and the Group Embedded Figures Test were used. Cognitive level for proportional logic was determined from a balance-beam task. A 42-item Geometry Achievement Test was constructed. The units studied focused on elementary concepts in geometry approached through transformations. The control classes did not study geometry until after the study ended. On the geometry posttest, the student-centered group had 80% of the items correct, compared to 68.6% for the teacher-centered group and 25.5% for the control group. Regression analysis indicated that students with high proportional logic ability achieved 5.5% more correct answers. Children with low spatial scores benefited more from the student-centered treatment. Other results were also presented. It was concluded that an inductive approach including active manipulation of objects helps foster understanding of transformations. (MNS)

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Spatial Ability and the Teaching of Introductory Geometry
Through Transformations

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Spatial Ability and the Teaching of Introductory Geometry
Through Transformations

The importance of children acquiring visual images for embodying the mathematics being learned has been acknowledged for some time. Bruner and Kenney (1965) suggest that learning begin "with instrumental activity, a kind of definition by doing " (p. 65). They further indicate:

Such operations become represented and summarized in the form of particular images. Finally, and with the help of a symbolic notation that remains invariant across transformations in imagery, the learner comes to grasp the formal or abstract properties of the things he is dealing with. (p. 59)

Richard Snow (1980) in a discussion of recent research in visual spatial imagery cites the creativity of theorists such as Einstein and the imaging ability of Seymour Cray, who mentally visualizes the internal makeup of computers before coming up with the design. He writes:

I have always thought that spatial imagery and visual metaphor, coupled with subsequent analytic reasoning processes, were at the heart of scientific and technical creativity. (p. 291)

Visual imagery, according to Snow, has been considered like spatial ability and sometimes interpreted as a cognitive style. Instruments that purport to measure these abilities are frequently used to distinguish high and low imaging ability. Clearly there are difficulties with the approaches used and Snow cautions us to be aware of the limitations of research in this area. He writes:

But a direct study of metaphoric thinking may also be dangerous to one's mental health. It forces us to face squarely the fact that all our theories are but metaphors at base--metaphors that we pursue, hoping dearly that they are apt. (p. 291)

One serious limitation of spatial abilities treatment studies appears to be the manner in which spatial treatment is defined. A treatment requiring the use of diagrams, for example, may be regarded as spatial. These diagrams, however, may not require the use of spatial reasoning. Further, whether spatial treatments do in fact require spatial abilities is still unresolved.

The present study sought to investigate whether instruction providing for experimental manipulation by students (student-centered) was superior to instruction in which teachers perform the manipulations and children observe (teacher-centered). Further, the study sought to determine whether spatial ability were related to mode of instruction, that is, do children with low spatial ability do better in a teacher-centered or student-centered instructional environment.

Theoretical Framework

The results of studies of the effectiveness of "alternate methods of instruction" have not always been consistent. Those reports that contrasted two or more treatments generally revealed that the teacher guided discovery method was at least effective as the others. In the teaching of geometry, studies conducted to compare a hands-on student activity approach and a teacher centered method of instruction showed conflicting results. Thomas (1976) and Corwin (1977) reported results that indicated no difference between treatments while Rowell and Mansfield (1980) found that a seven-lesson study with junior-high school students and introductory transformational geometry content favored a teacher-centered approach. Also, they found that achievement in geometry, independent of teaching method, was dependent on the student's

cognitive style.

Lynn Cooper (1980), using a process oriented approach to study spatial reasoning, suggests that there is a correspondence between subjects' mental rotations and external spatial rotations. She observed that mental rotations appear to follow a series of states that correspond to a series of stages occurring in the external rotation of objects. Her research indicates that not only do individuals differ in the processing strategies used but variations in the conditions under which an individual processes the spatial information yield differences.

Other approaches to measuring spatial ability have been psychometrically based. Cooper suggests that research utilizing a combination of both approaches has promise, citing as example, MacLeod, Hunt and Mathews' (1978) study in which a strategy for performance on a task was able to be predicted from a psychometric spatial ability measure.

Cronbach and Snow (1977), in their review of studies relating spatial ability to treatments that are "spatial", conclude that evidence points to a rejection of that relationship. They write:

Even though this chapter forces us to reject the conclusion that spatial treatments demand spatial ability, and the notion that the differentiated Guilford abilities will interact with "treatments of the same name", the door is not closed to positive findings in the future. Analysis of a certain set of lessons might suggest that certain specific internal manipulations are needed to encode and transform what is given. (p. 293)

Piaget (1973) stresses the role of experimental manipulations in the development of a child's understanding of the transformations of objects. He warns us that teacher demonstrations and audiovisual representations of transformations of objects are not equivalent to direct actions of

the learner and stresses the necessity for learners to form their own hypotheses and verify them through active manipulations of objects.

There has been considerable theoretical support for the view of learning as being contingent upon the activity and involvement of the learner. But how much of that activity needs to be hands-on for eighth graders? For 12 to 15 year old children, many of whom have not achieved formal operational thought, does the learning of elementary concepts in geometry approached through transformations of the plane require kinesthetic contact with the material? To what extent is their development of spatial abilities a factor in success? Specifically, do the the students with low spatial ability perform better in an instructional mode that provides for the manipulation of materials.

With these questions in mind, a study was undertaken to examine the effectiveness of two instructional treatments in the learning of basic concepts in geometry approached through transformations in the plane for students of varying spatial abilities and cognitive level in proportional reasoning. The lessons used for the eight-week study were identical except that in the teacher-centered mode, teachers performed the manipulations guiding the students through a series of questions and in the student-centered mode, the activities were performed by the students.

Objectives

The study had as its objectives the following:

1. to compare achievement of the student-centered, teacher-centered and control groups;
2. to examine whether spatial ability, proportional reasoning, teacher effects and prior knowledge of geometry are individually correlated with achievement;

3. to examine the effects of treatment (teacher-centered or student-centered), spatial ability (high or low cognitive restructuring and card rotations) and cognitive development (formal or non-formal proportional reasoning) on achievement and
4. to investigate ability by treatment interactions.

Data Source

The data were collected as part of a district program improvement project and a cooperative research program during the second year of the Rutgers University-Middlesex Public Schools Math Project. There were 141 twelve to fifteen year olds from seven eight-grade intact classes who participated in the study. Students were assigned to classes according to a policy that sought to group heterogenously. Analyses of achievement test scores, prior success in mathematics, and IQ scores supported the hypothesis of heterogeneity within and homogeneity among groups.

A randomized block design was used in which teachers were crossed with treatments to block out a teacher effect. The assignment of treatments and a control to each teacher resulted in Teacher 1 having one teacher-centered, one student-centered and one control class and Teacher 2 having one teacher-centered, two student-centered, and one control class. The control classes did not study geometry until after the study was terminated and instead studied computation of whole numbers and decimals.

Methods and Procedures

Spatial Ability

Two tests were used as measures of spatial ability, the Card Rotations Test and the Group Embedded Figures Test.

The Card Rotations Test comes from the Spatial Orientation Tests of the ETS Kit of Factor Referenced Cognitive Tests (Ekstrom, R., French, J., Harman, H. and Derman, D., 1976). The task requires that the subject select a form from a set of alternatives that can be derived by either a flip (mirror image) or a rotation of the given one. Subjects were given three minutes to complete each of two parts, consisting of ten items and containing eight forms. A total score of 160 points was possible. Test reliability averages 0.845 with variability due to the group tested.

The Group Embedded Figures Test was the second measure of spatial ability used in the study (Witkin, H., Oltman, P., Raskin, E. and Darp, S., 1971). The test, according to Witkin and Goodenough (1981) requires the subject to change the initial organization of the percept in order to perceive a target as a figure. To do so, they claim, requires the spatial manipulation of images and percepts.

The test consists of a set of eight simple figures, each of which is embedded in a complex figure. Subjects are asked to identify and trace the simple figure that is embedded in the complex one. As the task is being performed, both figures are not viewed simultaneously; simple figures were given on the back of the test booklet.

Field test results and information from a pilot study conducted the previous year suggested that students had insufficient time to complete the test. Hence, more time was given to students in accordance with recommendation of Witkin et al., (1971).

The test was scored according to the guidelines of the manual for Consulting Psychologist Press (1977). A total score of 18 points was possible.

Cognitive Level

Cognitive level for proportional logic was determined by evaluation of a subject's performance on a clinically-administered balance beam task and classified as formal or nonformal according to criteria described by Inhelder and Piaget (1958). The task, equilibrium of the balance, involves balancing a beam with two weights by taking into account the distance of each from the fulcrum. Thus an understanding of the inverse proportional relationship between weight and distance was necessary for success. Subjects successful on the task exhibit, according to Inhelder and Piaget (1958), the achievement of formal thought operations.

Prior to the study there was rigorous training of clinicians in preparation for administering the task. Clinicians approved to administer the task were required to score each of three subjects in separate interviews in accordance with more experienced clinicians. The agreement of scoring was independently decided by two evaluators. An agreement of 93 percent was achieved on the initial scoring for all clinicians participating in the study. Resolution of the remaining 7 percent was achieved by committee of six evaluators.

The selection of a task to measure proportional reasoning ability was made for two reasons. First, it was of interest to explore whether proportional reasoning level was related to activities involving symmetries and transformations of the plane. Second, significant influences of visual spatial ability on proportional reasoning were reported by Brendzel (1981) in a study conducted with high school students. Some measure of

whether students had attained formal operational logic on a task that called upon these spatial skills was desired.

Geometry Achievement

In the absence of a valid and reliable instrument to measure knowledge of concepts in geometry through transformations of the plane, a Geometry Achievement Test (GAT) was constructed on the basis of a set of twenty (20) concepts that were the object of instruction. A forty-two item test with 36 multiple choice questions and six construction problems was developed. It was first administered as a pre and posttest for a pilot study in the same school with children of the same age in the spring of the preceding year. Information from that pilot provided modification of the instrument.

Reviewers, ten math teachers and three mathematics-educators, examined the test for content validity, appropriateness of distractors in the multiple choice items, and clarity of questions; modifications were made where appropriate.

Students responded to the test items by circling the letter corresponding to the correct answer. For the six construction questions a five degree margin for error was permitted.

Each test was scored according to percent of items correct and then verified by two other reviewers. A reliability coefficient of 0.89 using the Kuder-Richardson Formula 21 was obtained.

The Curriculum

The units of study used over an eight-week period were the experimental pre-publication units of Math 2 (Maher, C., Rosenauer, W., Maher, J., Shay, K., Snyder, A., 1982) and were identical for both teacher-centered and student-centered classes. The published units are only slight refine-

ments of the originals and consist of elementary concepts in geometry approached through transformations of the plane. In teacher-centered classes, the activities of the units were performed by the teacher guiding the students through a series of questions to make generalizations. In the student-centered classes, materials, including concrete manipulatives, were provided to each student, making it possible for them to perform each activity. These materials, organized by Normandia (1981), included "miras", angle-fixers, rods, shapes, etc. The student-centered classes covered the same unit each day as did the activity-centered; however, within the time allotted, provision was made for individual differences in pace.

Results and Conclusions

Means and standard deviations of student scores on the pre and post-geometry achievement tests, spatial ability measures (card rotations and cognitive restructuring) and cognitive level for proportional logic are given in Table 1.

Insert Table 1 about here

For both the treatment and control groups, geometry pretest achievement assessments and spatial and cognitive level scores are similar. However, on the geometry posttest, the student-centered group had 80 percent of the items correct compared to 68.6 for the teacher centered group and 25.5 for the control group.

Table 2 contains the simple linear regression R^2 values of the percent variation in posttest achievement explained by the teacher effect, pretest score, and student ability measures.

Insert Table 2 about here

A multiple regression procedure was used to model the effect of the independent variables method of instruction (teacher-centered or student-centered), teacher (Teacher 1 or Teacher 2), geometry pretest score, spatial ability (card rotations or cognitive restructuring), and cognitive level in proportional logic (balance beam assessment) to the dependent variable, geometry posttest achievement and their interactions. The forward stepwise multiple regression was performed on the full three way interaction model using the SAS procedure (Barr, A., Goodnight, J., Sall, J., Blair, W. and Chilko, D., 1979), which adds main effects and interaction effects to the model one at a time with the most important effects first. Not all effects are added, only those that provide a significant improvement in the percent of variability in the dependent variable explained by the model. The results are given in Table 3 together with the multiple R^2 values (percent of variability in the dependent variable explained by the multiple regression model).

Insert Table 3 about here

The cumulative R^2 values are dependent on the order in which the variables were entered into the equation. The R^2 contribution of each effect is the Type IV SAS partial sum of squares which gives the contribution of the variable based upon the reduction to R^2 that would result if the variable were eliminated from the final regression model of Table 3. Thus, the R^2 contribution value is not dependent on the order in which the effects were added to the regression equation.

An interpretation of the regression coefficients of Table 3 is that students with high proportional logic ability achieved 5.52 percent more correct answers on the geometry posttest than those with

low ability and students taught by Teacher 1 averaged 4.52 percent more correct answers than those taught by Teacher 2. For the regression equation, the variable that represents geometry pretest achievement has a coefficient of 0.23.

The treatment $\times$ card rotation $\times$ cognitive restructuring ability interaction is given by Expression 1 where A represents the student's cognitive restructuring ability, C represents card rotation ability and B is defined as 1 for the student-centered treatment and 0 for the teacher-centered.

$$2.21 A + 15.37 B + 0.21 C - 0.008 A \cdot B \cdot C \quad (1)$$

The posttest interaction effect is illustrated in Figure 1.

Insert Figure 1 about here

The graph indicates that the student-centered mode of instruction resulted in significantly higher posttest scores for students who were below average in either card rotation or cognitive restructuring ability.

Figure 2 provides a graph of the difference between the interaction effect of the activity-centered method of instruction.

Insert Figure 2 about here

Students in the lower 25th percentile range in either cognitive restructuring or card rotations scored on the average 10 to 15 points higher when taught by the student-centered method. However students in

the upper 10 percent range on both spatial tasks performed, on the average, 1.2 to 5.1 points higher when taught by the teacher-centered approach.

Discussion

The results of this study support the view that an inductive approach to instruction that includes experimenting with objects by actively manipulating them helps foster the development of the child's understanding of the transformations of objects.

The consistent difference in performance of Teacher 1's classes over Teacher 2's can be explained by the fact that Teacher 1 had considerably more training in mathematics and experience teaching it.

The study also lends support to the research finding of Rowell and Mansfield (1980) of cognitive level as a significant main effect without a treatment by cognitive level interaction. However, by including measures of spatial ability, this research provides a more refined understanding of the relationship of student ability and achievement in geometry approached through transformations with teacher-centered and student-centered modes of instruction.

The fact that children with low spatial scores most benefited from the student-centered treatment suggests that the "instrumental activity" and "learning by doing" in the Bruner sense was helpful. Perhaps not having immediately to imagine but rather first to be able concretely to build an operation served to facilitate their learning. The children with high spatial scores may have already possessed the imagery needed for the mathematics being presented by the teacher. Consequently, the experiences provided by the student-centered mode may not have been necessary for their learning the concepts involved.

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Table 1

Means and Standard Deviations for Selected Variables

Variable	Maximum Score	Treatment					
		Student Centered (n=58)		Teacher Centered (n=48)		Control (n=35)	
		Mean	SD	Mean	SD	Mean	SD
Card Rotations	160	79.0	32.3	79.6	31.3	68.7	29.0
Cognitive Re- structuring	18	11.0	4.5	9.3	4.9	10.7	4.7
Proportional Logic	F ^a	34.5 ^b	-	30.2	-	39.4	-
Geometry Pretest	100	18.7	8.7	18.8	9.1	17.1	7.9
Geometry Posttest	100	80.0	12.2	68.6	14.5	25.5	12.5

^aCognitive level was assessed as formal or nonformal based upon performance on the balance beam task.

^bPercent of students assessed as formal

Table 2

Simple Linear Regression R^2 Values for Predicting Geometry
Posttest Achievement

Independent Variable	R^2
Cognitive Restructuring	0.27
Treatment	0.15
Card Rotations	0.14
Teacher	0.09
Proportional Logic	0.07
Geometry Pretest	0.04

Table 3

Multiple Linear Regression Model for Predicting Geometry Posttest Achievement

Variable	Regression Coefficients	STD Error	Cumulative R^2	R^2 Contribution ^a	F	Prob > F
Cognitive Restructuring (A)	2.21	0.47	26.8	12.8	22.32	0.0001
Treatment (B)	15.37	3.43	36.1	12.0	20.91	0.0001
Card Rotations (C)	0.21	0.06	41.2	7.8	13.66	0.0004
A × B × C Interaction	-0.0085	0.003	45.3	4.6	8.13	0.0055
Proportional Logic (D)	5.52	2.39	48.4	3.1	5.34	0.0233
Teacher (E)	4.52	2.34	50.7	2.1	3.72	0.0572
Geometry Pretest (F)	0.23	0.13	52.5	1.8	3.07	0.0835
Intercept	12.77	-	-	-	-	-

^aContribution is based upon the reduction in R^2 that would result if the factor were eliminated from the final multiple regression model.

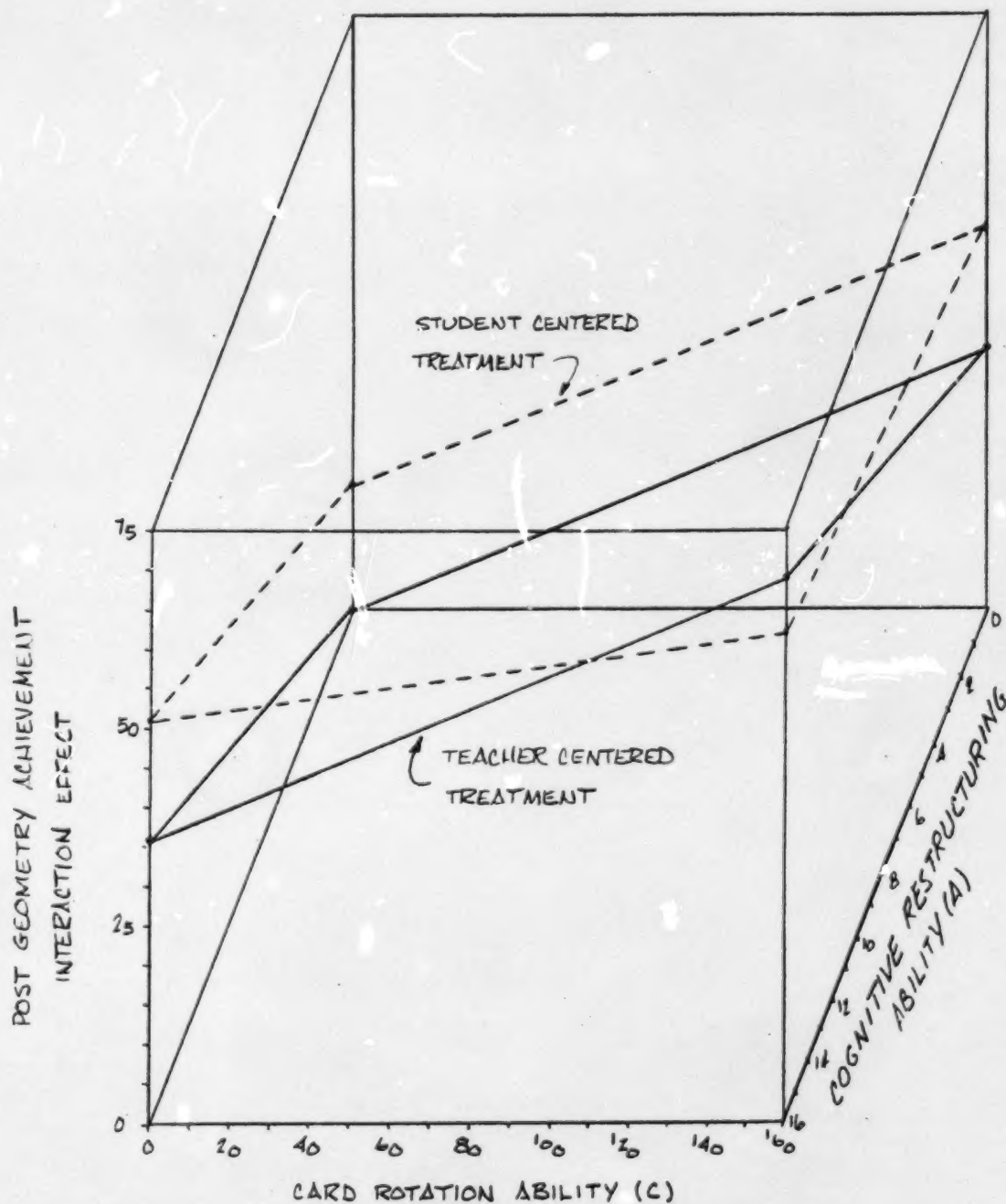


Figure 1. Treatment $\times$ card rotations $\times$ cognitive restructuring interaction effect on geometry posttest achievement.

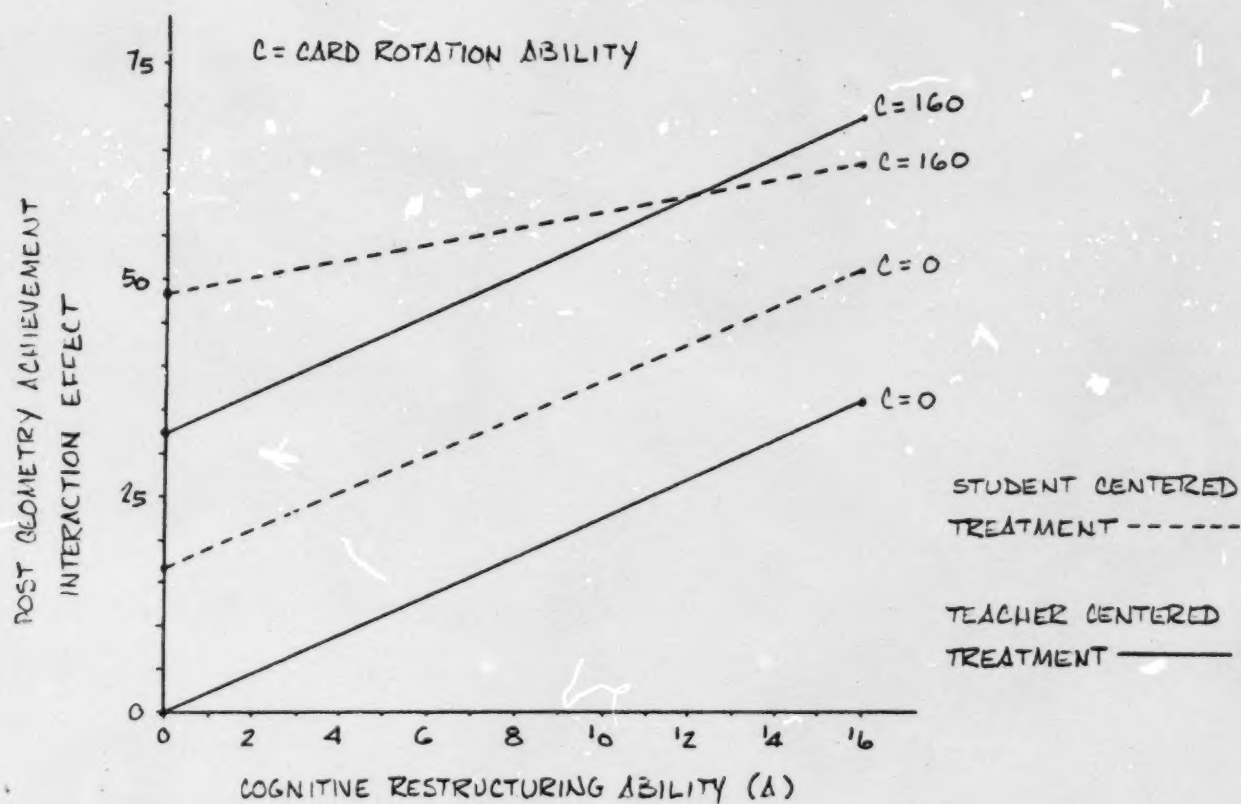


Figure 2. Treatment $\times$ card rotations $\times$ cognitive restructuring interaction effect on geometry posttest achievement for extreme values of card rotations.